Adaptation and survivors in a random Boolean network

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We introduce the competitive agent with imitation strategy in a random Boolean network, in which the agent plays a competitive game that rewards those in minority. After a long time interval, the worst performer changes its strategy to the one of the best and the process is repeated. The network, initially in a chaotic state, evolves to an intermittent state and finally reaches a frozen state. Time series of survived species (whose strategies are imitated by other agents) in the system depend on the connectivity of each agent. In a system with various connectivity groups, the low connectivity groups win the minority game over the high connectivity groups. We also compared the result with mutation strategy system.

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In the agent based models used in social sciences, economy, ecosystems, and genetic regulatory systems, agents interact directly with others, and global structures emerge from these interactions. An important question that is being addressed in a number of ways is how the aggregate or global behavior emerges from the individual characteristics of the agents. A particular aspect of this question is to understand whether the global behavior is determined by average commonly found agents or if a few individual distinct agents can have a strong influence on the macrostructure. In the latter case such special agents play the role of leaders.

Generally speaking, the influence of each agent depends on the network of interactions with other agents. Each agent's strategy can be represented as a function that specifies a set of outputs for each possible input. Random Boolean networks (RBN) proposed by Kauffman [1] are classic discrete genetic models where agent's strategy is represented as a simple Boolean function. It is shown that global order is able to emerge from local rules in RBN. The state of an agent can be represented by two possible values (1 or 0). This state is the output of a Boolean function, which has as inputs the output activity of some other agents. The connectivity of the system and the bias used for the Boolean functions are relevant parameters in order to statistically determine the network dynamics. In most cases, the network of interactions is fixed and given from the outset. However, it is natural to consider the situation in which the network of interactions evolves dynamically adapting itself to the global structure. Adaptive system can be roughly characterized by two elements, one who should adapt and the other how to adapt itself. As a primary step towards adaptive systems, Challet and Zhang [2] proposed the so-called minority game (MG) in which an odd number of agents successively compete to be in the minority. There have been many studies of the statistical properties of MG, which treat the game as a quasistochastic system [3–5]. In a recent paper, Paczuski, Bassler, and Corral [6] introduced RBN with minority win game. The agents are competing against each other and at each time step those in the minority win. The agent who was in the majority most often over a long time scale, the epoch, changes its strategy randomly (mutation). They observed that the network eventually evolves to a stationary but intermittent state. The change of strategies is approximated as an extremal process [7]. In a natural complex system, typical ways of adaptation are imitation and mutation.

Hence, motivated by these works, we tackle the problem of how cooperation arises in a dynamically evolving network of agents. We have investigated how leading agents emerge in the RBN by introducing worst performer's imitation strategy [8]. The best performer whose strategy is imitated by other agents may play the role of leader.

We consider a network of N (odd) agents where each agent is assigned a Boolean variable $\sigma_i = 0$ or 1. The N agents separate, in general, into subcategories of size $N_1, \ldots, N_m$ with $\Sigma_{k=1}^{m} N_k = N$. A subcategory N_k has a connectivity of k. Each agent i receives input from K_i other distinct agents chosen at random in the system. The set of inputs for each agent i is quenched. The evolution of the system is specified by N Boolean functions of K_i variables, each of the form at time $t+1$,

$$\sigma_i(t+1) = f_i(\sigma_{i_1}(t), \sigma_{i_2}(t), \ldots, \sigma_{i_{K_i}}(t)). \tag{1}$$

Each Boolean function f_i is chosen from possible $2^{2^{K_i}}$ functions. It can be characterized by a homogeneity parameter p, which represents that the value 1 is assigned to an output with a probability p and 0 with a probability $1-p$. With specified initial conditions, random but biased by a homogeneity p, each agent is updated in parallel according to Eq. (1). The control parameters (K_i and p) determine two regions: a frozen phase and a chaotic phase. They exhibit a phase order-disorder transition modulated by the values of their control parameters. In the single connectivity system (all agents have connectivity K), the critical condition is represented as follows:

$$K_c(p) = \frac{1}{2p(1-p)}. \tag{2}$$

For $K < K_c$ RBN starting from random initial condition reaches frozen phase, while $K > K_c$ RBN reaches chaotic phase. RBN with $K = K_c$ are critical and the distribution of attractor lengths that the system reaches, starting from random initial conditions, approaches a power law for large enough system sizes consistent with all previous work on Kauffman networks.

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In our model, the agents in the minority are given their individual score at each time step. The network was updated until either the attractor of the dynamics was found, or the length of the attractor was found to be larger than some limiting value which was typically set at 10000 time steps solely for reasons of numerical convenience. For large K, the attractor length is found to scale approximately as $\sqrt{2^N}$ [9]. As one epoch is terminated, the worst performer who has the least score in the system is chosen for "imitation" selection. The worst performer chose the input nodes and output Boolean table the same as the best performer, substituting

$$f_n(\sigma_{n_1},\sigma_{n_2},\ldots,\sigma_{n_{K_n}})\rightarrow f_m(\sigma_{m_1},\sigma_{m_2},\ldots,\sigma_{m_{K_m}}), \quad (3)$$

where index m (n) is the worst (best) performer; that is, the worst performer imitated the strategy of the highest scores. If two or more agents are the worst (best) performers, one of them is chosen at random and changed. Starting from a random state, where K is above the critical value K_c, the network is initially driven into the chaotic phase. However, as epoch goes on, it becomes difficult to reside in the chaotic phase, because the imitation strategy decreases monotonically the number of species in the system. If all the agents begin to use similar strategies, and hence make the same decision, such a strategy ceases to be profitable. Therefore, any particular strategy's success is short-lived. An emergence of leader agents corresponds to a decrease of survived species. The species are defined as the bunch of agents who have the same internal function and the same input agents. The agent who is imitated by others the most number of times is the most prominent leader in the network system. We found that the emergence of leaders is closely related to the connectivity in the system by observing epoch series (defined as T) of survived species ratio n_{sv}, actual average homogeneity in the system P, and attractor length.

We first investigated a single connectivity with initially no bias system (initial actual average homogeneity $P_{init}=0.5$) where all agents have the same connectivity value K, that is, $N_K=N$, other $N_k(k\neq K)=0$. The process of adaptation is monitored by measuring several parameters, such as survived species, homogeneity, scores of the minority game for agents, distributions of agents' past records of best and worst performers. The best (worst) performer's distribution can be characterized by the number of times of selections $B_i(W_i)$ for an agent i. In order to estimate network dynamical behavior, it is instructive to define dynamical parameters, such as unitary percent of agent's self-correlation $a(t,s)$ in time t and $t+s$ [10], average activity of an agent i: $A(i)=1/(t_2-t_1)\Sigma_{t=t_1}^{t_2}\sigma_i(t)$, where the sum is taken over the dynamical attractor defined by t_1 and t_2 [11,12].

We have simulated the network system with $N=441$ as long as 10^5 epoch steps. The evolution of $n_{sv}(K,T)$ for various values of K is shown in Fig. 1. For all K's, T dependence of $n_{sv}(K,T)$ is as follows:

$$n_{sv}(K,T)=\exp\{-\alpha(K)T/N\} \quad \text{if} \quad T\ll T_c(K),$$

$$n_{sv}(K,T)=\text{const}\times T^{-\beta(K)} \quad \text{if} \quad T\gg T_c(K). \quad (4)$$

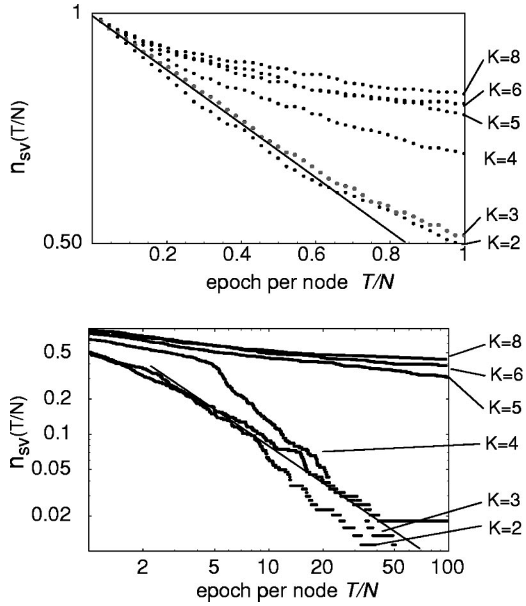

FIG. 1. Evolution of the survived species with epoch for various values of K and $N=441$ and $P_{ini}=0.5$. To avoid trapping in exponential divergence of attractor length for high K, the simulations have been limited to $t_{max}=1000$. No averaging has been performed on the data. (a) Distribution of survived species (in log-linear plot) for $K=2,3,4,5,6,8$. The fraction of survived agents appears to saturate, it strays off the plot by increasing epoch steps further. (b) Distribution of the survived species (in log-log plot) for $K=2,3,4,5,6,8$ and $T/N\geqslant 1$.

Critical epoch T_c is defined as an epoch at which the survived species of the system change from exponential distribution to power-law distribution over epoch steps.

For $K\geqslant 5$, which we define as a high K region, the network system in the initially chaotic state never reached the intermittent state, neither frozen state. $n_{sv}(K,T)$ is initially exponential distributed with $\alpha\simeq 7.7\pm 0.1\times 10^{-1}$ and strays off the plot after epoch steps $T/N\simeq 0.13$.

The system resides in a "diversity" state in which many species could survive for a long period of time. A fit with points gives $\beta=0.17\pm 0.03$. $n_{sv}(K,T)$ in the high K region is expected to reach 1% of initial state in 10^{12} epoch steps, where as in the low K region, it always reached within 10^5 epoch steps. The standard deviation of the worst performer's selection distribution W for $K=6$ averaged over 44 100 epoch steps shows $\sigma_W/\langle W\rangle\simeq 0.106$, compared to that of RBN minority game with the random update strategy $\sigma_{W^R}/\langle W^R\rangle\simeq 0.135$. This result indicates that the difference between the best performer and the worst one can be shrunk by introducing the imitation strategy in the chaotic state. For $3\leqslant K\leqslant 4$, which we define as low K region including $K\leqslant 2$, independent of the initial connectivity and Boolean value, the network system in the initially chaotic phase, attractor length being very long, evolves to an intermittent state. Furthermore, it finally evolved to a frozen state, and never evolved to an intermittent state again. The system finally evolves to a "unified" frozen state within $T/N\simeq 50$ in which less than 1% of initial species can survive the game. The typical picture of intermitting attractor arising from this model is shown in Fig.

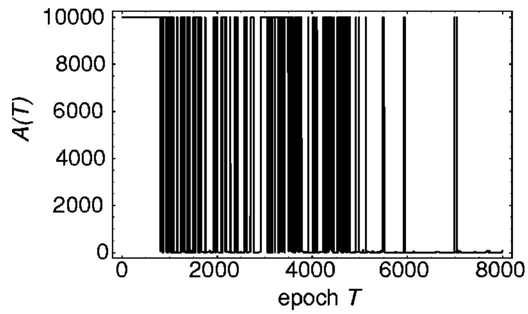

FIG. 2. Time series of attractor length in each epoch for $K=3$, $N=1089$, and $P_{ini}=0.5$ in the stationary state. The system evolves to the intermittent state in epoch 812, and finally reaches the frozen state in epoch 7035.

2 for a system of $K=3$, $N=1089$, $p_{ini}=0.5$. In comparison to the random update strategy shown in Fig. 1 of [6] with the same connectivity $K=3$, it takes less time to reach the intermittent state and reach the frozen state. This is because the conformity among the same species may act as a freezing element to the network. In Fig. 1, the case shown in the system evolves to the intermittent state at epoch per nodes $T/N \simeq \{0.51, 5.18\}$ and reaches the frozen state at epoch per nodes $T/N \simeq \{3.78, 17.3\}$ for $K=\{3,4\}$. The coefficient β changes when the system evolves to the intermittent state. For $K \leqslant 2$, the network system, initially in the frozen state, remains in the same state with rapid decrease of survived species as epoch step increases. It finally reached "unified" state in which only 1% of initial species or less could survive the game. A fit with points gives exponent $\alpha \simeq 7.7 \pm 0.1 \times 10^{-1}$ for initial epoch steps. After several epoch steps $T/N \simeq 0.7$, the evolution of $n_{sv}(K,T)$ saturates around $n_{sv} \simeq 0.60$ and turns into power-law distributed with exponent $\beta \simeq 0.9$. Critical connectivity K_c that distinguishes between "unified" and "diversity" states may depend on N, and is between $4 < K < 5$ above the critical K of Eq. (2). The initial state and randomness of selections have an influence on determining the ratio of decrease.

We also observed fluctuation of average homogeneity of all agents P over epoch steps. In Fig. 3, we plot the evolution of P for both $K=3$ and $K=6$ generated by a game with $N=441$, $P_{ini}=0.5$. For $K=\{2,3,4,5,6,8\}$, standard deviation of homogeneity averaged over 10 000 epoch steps shows $\sigma_p \simeq \{1.6 \times 10^{-2},\ 7.3 \times 10^{-3},\ 9.3 \times 10^{-3},\ 1.7 \times 10^{-3},\ 8.5 \times 10^{-4},\ 5.3 \times 10^{-4}\}$. On the whole, the fluctuation of P increases as the ratio of the frozen state is increased, that is, as K is smaller. On the other hand, the distribution of W_i, $D_i = B_i - W_i$ shows no distinct difference between high K and low K that standard deviation σ_D is almost independent of K, $\sigma_D \simeq 14.5$ averaged over ten times for each K. The randomness of the best (worst) performer's selection becomes higher as the attractor length is getting shorter, since more agents would become the candidate for the best (worst) performer. However, the histogram for both high and low K shows the similar probability distribution.

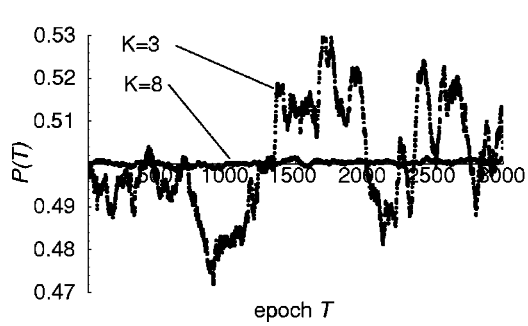

FIG. 3. Time series of homogeneity P for $K=3$, $K=8$ ($N=441$, $P_{ini}=0.5$).

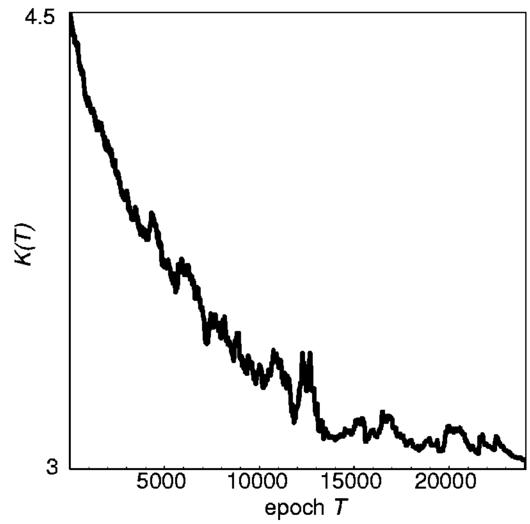

FIG. 4. Time series of average connectivity K_{av} of two groups RBN minority game in a particular simulation for $N=1089$. The initial states (at $T=0$) are $N_3=545$, $N_6=544$, and $P_{ini}=0.5$. All agents that have $K=6$ have died out at epoch 24 071, and all 23 species that have survived the game have $K=3$.

We have simulated the size dependence of $n_{sv}(K,T,N)$ with various system sizes N and find the functional form $n_{sv}(K,T/N)$ is independent of N as far as the system is in the low K region. But for smaller N, there is some possibility that the system of high K region could evolve to an intermittent state and to a frozen state as the degree of freedom is reduced. For example, the initially chaotic system evolves to frozen state three times out of ten with $K=5$, $N=121$ within $T/N < 10$ epoch steps. To determine the primary factor that determines the evolution of the system is still an open issue.

The same transition from unified state to diversity state could be found when we vary p instead of K. For fixed $N=441$, $K=6$, the transition would occur when P_{ini} is below $P_c \simeq 0.64$. The initially biased system may evolve to $P \simeq 0.5$ as epoch step increases. A freezing action of decreasing species and a chaotic action of evolving P compete with each other, which may determine the evolved state of the system.

Second we investigated the no biased ($P_{ini}=0.5$) RBN minority game with the same imitation rules where agents with various connectivity values are mixed. Figure 4 shows average connectivity $K_{av} = 1/N \Sigma_{k=1}^{m} k N_k$ versus the initial fraction of the low connectivity agent, where there are two groups of agents, $N_6 \neq 0$ (as a representative of chaotic state), $N_3 \neq 0$ (as of frozen state), $N_i(i \neq 3,6)=0$, where a group is defined as an agent's herd with same connectivity value. The figure shows that the two groups compete, and the low K group gains an advantage over the high K group to win the minority game. Even when the fraction of low connectivity $r=0.1$ in the initial state, most of agents in the system finally evolve to those with smaller connectivity in several epoch steps. If the homogeneity is biased in the system, it is natural that the agent with low connectivity have a higher probablity to win the game. They have a high probability to be in all 0 or all 1 state. Hence, they are apt to be selected as the best/worst performer in a constant output zeros or 1's winning system. Even in the no-biased state, we observe that a small group with low K wins the Minority game over the high K group. Although low ones are vulnerable to the selection in the same group. In general, with low connectivity the agents have a higher probability of survivals. We also obtained the same result from the system of which initial group distribution is $N_1=N_2=\cdots=N_{10}$.

In summary, we have studied the minority game of random Boolean networks and their topological evolution on the basis of the worst performer imitation rules. The imitation rules give rise to the emergence of the leader who have its strategy in its origin, and cooperators who imitate leader's strategies. The connectivity parameter determines two regions of survival species: unified phase and diversity phase. We have observed that in unified phase, the number of species decreases rapidly and a few species remain in the system. On the other hand, in diversity phase, it becomes power-law distributed with a small coefficient and quite an amount of species could remain for a long time. In a network system with various connectivity groups, we have numerically shown that the group with low K is more likely to survive the minority win game. Much future work is open to all sorts of variation. An interesting question is whether a comparable mechanism may occur in a natural complex system. One example where such mechanism could occur is the regulation of buying/selling activity in the market [13]. In order to consider social and marketing implementations, we should consider agents with mutation rules and imitation rules altogether, which may be essential features for the dynamics of the complex network system. We should also include several rules that allow different types of updated for each agent to implement more actual social/economic model. We still have no theoretical description even for the primitive imitation model. To explain that the system may be strongly influenced by the initial states and random choice of the best (worst) performers when the parameter approaches the critical value, and whether the dynamics can be explained in terms of average parameters or leading particular ones, is a critical issue.

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